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Finding Optimal Production and Selling Strategies for an Electricity Generator in a Part of a Country's Electrical Grid

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Abstract

A part of a country's electrical grid in which an electricity generator (which may consist of several base load power plants and several peaking power plants) supplies electricity to a set of large customers of the grid, whereas the customers can a) receive electricity from renewable sources of energy, b) store electricity in certain volumes, and c) buy electricity in the markets is considered. It is proposed to describe the interaction of the generator, the large grid customers, and the transmission company (under uncertainty of the customer demand for electricity) by a game with a finite (more than three) number of players on polyhedra of player strategies some of which are connected and thus cannot be chosen by the players independently of each other. Sufficient conditions for the game equilibria verifiable by solving three linear programming problems are proposed, and the equilibria particularly determine optimal production and selling strategies for the generator.

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Keywords: Electrical grid; electricity storages; equilibrium points; linear programming; *n*-person game with connected player strategies; three-person game with disjoint player strategies.

1. Introduction

Let us consider a part of an electrical grid of a country in which an electricity generating facility, which may consist of several base load power plants and several peaking power plants, all acting as one legal entity (called the generator further in this article), supplies electricity to a set of large customers of the grid [1]. This set consists of

a) companies providing electricity for individual end users in industrial and residential areas (households) and small enterprises and businesses via (low voltage) distribution lines (the utility companies),

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b) industrial enterprises and large businesses that receive electricity from the generator under direct supply agreements (the industrial customers),

c) groups of end users that are licensed to operate the (low voltage) distribution lines via which electricity is delivered to them directly, rather than via utility companies (the groups of advanced customers).

These grid customers and the generator interact via a company (or a group of companies acting as one legal entity) that delivers (transmits) electric energy from the generator to these customers via (high voltage) transmission lines (the transmission company).

All the above customers can receive electricity from renewable sources of energy, as well as can buy electricity in electricity markets, and each of these customers either possesses or has access to a fleet of electricity storage facilities into which electricity comes from the generator, from renewable sources of energy (via devices transforming different kinds of energy into electricity), and from the electricity markets. For the sake of definiteness, only wind and solar sources of energy are considered as renewable ones further in this article, and it is assumed that electricity from these sources of energy is exclusively provided by the companies, called suppliers further in this article, via electricity storage facilities that are rented by these suppliers to the large grid customers.

The transmission company charges both each large grid customer (for the volumes of electricity that this customer receives from the generator) and the generator (for the volume equaling the sum of the electricity volumes delivered to the customers and the volume of electricity lost in transmitting electricity to them).

The generator is interested in finding at which prices it should supply electricity to the above grid customers under potential long-term contractual agreements, taking into account a) its capacity to produce electricity and its chances to sell electricity in the markets under expected prices there, and b) its expenses associated with the electricity production and transmission. It is also interested in determining an optimal hourly production schedule (i.e., a constant hourly volume of electricity to be produced within every twenty-four-hour period of time) corresponding to the above contractual agreements, taking into account the range of an expected demand for each customer and the ability of the customers both to receive electricity from renewable sources of energy and to buy it in the markets [1-2].

In the present article, it is assumed that all the grid customers can buy electricity in the markets and have this electricity delivered to them by a transmission company that may not necessarily be the one that delivers electricity to them from the generator. These two assumptions are additional ones to those under which the results in [1-2] were obtained. However, despite obvious differences between the model of the grid functioning presented in [1-2] (and reflecting the assumptions made there) and the one under consideration in this article, the basic result from [1-2]–the sufficient conditions for equilibria in the game describing the functioning of the part of the gird, whose verification consists of solving three linear programming problems two of which form a dual pair [2]–remains true for the problem under consideration in this article.

2. The mathematical formulation of the problem

To estimate an optimal production schedule for a generator in a part of a country's electrical grid, a mathematical description of the functioning of all the large grid customers, along with that of the generator and the transmission company, is needed. This description should be based on the generator knowledge about the customer potential to consume electricity and to receive it from all the available sources, as well as on that about the prices for electricity in the markets available to the large grid customers, about the prices for electricity from renewable sources of energy, and about the prices for transmitting electricity within the region. To describe the above functioning by constraints of the balance kind that bind the volumes of the electricity received, stored, and consumed with the electricity prices and the transmission cost, sets of variables and parameters are to be introduced.

To describe the functioning of industrial customer i, $i \in \overline{1,m}$, let

$y_i^w(l)$ be the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from wind energy in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{w-st}(l)$ be a part of the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from wind energy and that goes to the storage system of this customer in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{w-dir}(l)$ be a part of the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from wind energy and that is used by this customer directly, beginning from the moment of receiving this energy in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^s(l)$ be the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from solar energy in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{s-st}(l)$ be a part of the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from solar energy and that goes to the storage system of this customer in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{s-dir}(l)$ be the part of the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from solar energy and that is used by this customer directly, beginning from the moment of receiving this energy in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$(yM)_i(l)$ be the volume of electric energy that has been bought by industrial customer i, $i \in \overline{1,m}$, in an electricity market (or in several electricity markets) and that is to be received by this customer in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$(yM)_i^{st}(l)$ be the volume of electric energy that has been bought by industrial customer i, $i \in \overline{1,m}$ in an electricity market (or in several electricity markets) and that is to go to the storage system of this customer in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$(yM)_i^{dir}(l)$ be the volume of electric energy that has been bought by industrial customer i, $i \in \overline{1,m}$, in an electricity market (or in several electricity markets) and that is to be received and used by this customer directly in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^g(l)$ be the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from the generator,

$y_i^{g-dir}(l)$ be a part of the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from the generator and that is used by this customer directly, beginning from the moment of receiving this energy in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{g-st}(l)$ be a part of the volume of electric energy that is received by industrial customer i, $i \in \overline{1,m}$ from the generator and that goes to the storage system of this customer in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{st}(l)$ be the volume of electric energy available to industrial customer i, $i \in \overline{1,m}$ from its storage system in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{st}(0)$ be the volume of electric energy in the storage system of industrial customer i, $i \in \overline{1,m}$ at the beginning of the twenty-four-hour period of time,

$y_i^{st+}(0)$ be the maximum volume of electric energy that can be stored in the storage system of industrial customer i, $i \in \overline{1,m}$,

$\underline{y}_i^{st}$ be the minimal hourly volume of electric energy that is to be maintained by industrial customer i, $i \in \overline{1,m}$ in its storage system,

Δ_i^y be the hourly average energy loss associated with storing electricity in the storage system of industrial customer i, $0 < \Delta_i^y < 1$, $i \in \overline{1,m}$,

λ_i^{yw} be the (average hourly) expenses of industrial customer i, $i \in \overline{1,m}$ that are associated with receiving a unit volume of electric energy from wind energy,

λ_i^{ys} be the (average hourly) expenses of industrial customer i, $i \in \overline{1,m}$ that are associated with receiving a unit volume of electric energy from solar energy,

π_i^y be the (average hourly) expenses of industrial customer i, $i \in \overline{1,m}$ that are associated with operating its storage system with the storage capacity equaling a unit volume of electricity stored,

$\underline{y}_i^w(l)$ be the estimate of the minimal volume of electricity produced from wind energy that industrial customer i, $i \in \overline{1,m}$, agrees to receive from the corresponding supplier (s), $l \in \overline{1,24}$,

$\underline{y}_i^s(l)$ be the estimate of the minimal volume of electricity produced from solar energy that industrial customer i, $i \in \overline{1,m}$, agrees to receive from the corresponding supplier (s), $l \in \overline{1,24}$,

$y_i^{dem}(l)$ be the electricity demand of industrial customer i, $i \in \overline{1,m}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$\overline{y}_i^{dem}(l)$ be the estimate of the maximal electricity demand of industrial customer i, $i \in \overline{1,m}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$\underline{y}_i^{dem}(l)$ be the estimate of the minimal electricity demand of industrial customer i, $i \in \overline{1,m}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$,

$y_i^{dem-st}(l)$ be the volume of electric energy that is consumed by industrial customer i, $i \in \overline{1,m}$ from its storage system in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$.

The above constraints of the balance kind describing the functioning of industrial customer i, $i \in \overline{1,m}$ and binding all the listed variables and parameters look as follows:

$$
\begin{aligned}
&\underline{y}_i^{w}(l) \le y_i^{w}(l),\\
&\underline{y}_i^{s}(l) \le y_i^{s}(l),\\
&\underline{y}_i^{dem}(l) \le y_i^{dem}(l) \le \overline{y}_i^{dem}(l),\\
&y_i^{st}(l) = y_i^{st}(0) + \sum_{x=1}^{l} (y_i^{g-st}(x) + y_i^{s-st}(x) + y_i^{w-st}(x) + (yM)_i^{st}(l)) - \sum_{x=1}^{l} (y_i^{dem-st}(x) + \Delta_i^y x),\\
&\underline{y}_i^{st} \le y_i^{st}(l) \le y_i^{st^+},\\
&y_i^{dem}(l) = y_i^{w-dir}(l) + y_i^{s-dir}(l) + y_i^{g-dir}(l) + (yM)_i^{dir}(l) + y_i^{dem-st}(l),\\
&y_i^{w}(l) = y_i^{w-dir}(l) + y_i^{w-st}(l),\\
&y_i^{s}(l) = y_i^{s-dir}(l) + y_i^{s-st}(l),\\
&y_i^{g}(l) = y_i^{g-dir}(l) + y_i^{g-st}(l),\\
&(yM)_i(l) = (yM)_i^{dir}(l) + (yM)_i^{st}(l),\\
&i \in \overline{1,m},\ l \in \overline{1,24}.
\end{aligned}
\tag{1}
$$

Here, it is assumed that system (1) is compatible, which can be verified by a simple technique, proposed in [3].

Further, let

$p_i^y(l)$ be the price at which a unit volume of electric energy is sold by the generator to industrial customer i, $i \in \overline{1,m}$ in the period of time from hour $l-1$ to hour l, $l \in \overline{1,24}$.

The functioning of the other large grid customers can be described by similar systems of constraints [1] binding variables and parameters that have the same meaning for utility company j, $j \in \overline{1,n}$ and for group of advanced customers k, $k \in \overline{1,r}$ as do the (corresponding to them) variables and parameters that are used for describing the functioning of industrial customer i, $i \in \overline{1,m}$

$z_j^w(l),\ z_j^{w-st}(l),\ z_j^{w-dir}(l),\ z_j^s(l),\ z_j^{s-st}(l),\ z_j^{s-dir}(l),$

$(zM)_j(l),\ (zM)_j^{dir}(l),\ (zM)_j^{st}(l),\ z_j^g(l),\ z_j^{g-dir}(l),\ z_j^{g-st}(l),\ z_j^{st}(l),$

$z_j^{dem-st}(l),\ z_j^{st}(0),\ \underline{z}_j^{st},\ z_j^{st^+},\ \Delta_j^z,\ \lambda_j^{zw},\ \lambda_j^{zs},$

$\pi_j^z,\ z_j^{dem}(l),\ p_j^z(l),\ \overline{z}_j^{dem}(l),\ \underline{z}_j^{dem}(l),\ \underline{z}_j^w(l),\ \underline{z}_j^s(l),\ j \in \overline{1,n},\ l \in \overline{1,24}.,$

and

$u_k^w(l),\ u_k^{w-st}(l),\ u_k^{w-dir}(l),\ u_k^s(l),\ u_k^{s-st}(l),\ u_k^{s-dir}(l),$

$(uM)_k(l),\ (uM)_k^{dir}(l),\ (uM)_k^{st}(l),\ u_k^g(l),\ u_k^{g-dir}(l),\ u_k^{g-st}(l),\ u_k^{st}(l),$

$u_k^{dem-st}(l),\ u_k^{st}(0),\ \underline{u}_k^{st},\ u_k^{st^+},\ \Delta_k^u,\ \lambda_k^{uw},\ \lambda_k^{us},$

$\pi_k^u,\ u_k^{dem}(l),\ p_k^u(l),\ \overline{u}_k^{dem}(l),\ \underline{u}_k^{dem}(l),\ \underline{u}_k^w(l),\ \underline{u}_k^s(l),\ k \in \overline{1,r},\ l \in \overline{1,24}.$

Further, let [1]

θ^y be the vector whose component θ_i^y is the price for a unit volume of electric energy that the transmission company charges the generator for the use of the transmission lines to transmit electricity to industrial customer i, $i \in \overline{1,m}$ and it is assumed that this price remains the same for the generator during every hour within the twenty-four-hour period of time,

s^y be the vector whose component s_i^y is the price that industrial customer i as a customer of the part of the electrical grid pays the transmission company for a unit volume of electric energy transmitted to this industrial customer from the generator, $i \in \overline{1,m}$, and it is assumed that this price remains the same for this industrial customer during every hour within the twenty-four-hour period of time,

θ be the price for a unit volume of the electric energy lost in transmitting electricity to the grid customers via the (high voltage) transmission line at which the transmission company charges the generator, and it is assumed that this price remains the same for the generator during every hour within the twenty-four-hour period of time,

and let the numbers $\theta^z, \theta_j^z, s^z, s_j^z$ and the numbers $\theta^u, \theta_k^u, s^u, s_k^u$ have the same meaning for utility company j, $j \in \overline{1,n}$ and for group of advanced customers k, $k \in \overline{1,r}$ as do the numbers $\theta^y, \theta_i^y, s^y, s_i^y$ for industrial customer $i \in \overline{1,m}$, respectively.

Finally, let [1]

$$y_i^g = (y_i^g(1),\ldots,y_i^g(24));\ y_i^w = (y_i^w(1),\ldots,y_i^w(24));\ y_i^s = (y_i^s(1),\ldots,y_i^s(24));$$

$$p_i^y = (p_i^y(1),\ldots,p_i^y(24));\ \lambda_i^{yw}(av) = (\lambda_i^{yw},\ldots,\lambda_i^{yw});\ \lambda_i^{ys}(av) = (\lambda_i^{ys},\ldots,\lambda_i^{ys});$$

$$y_i^{st} = (y_i^{st}(1),\ldots,y_i^{st}(24));\ \lambda_i^{yw}(av),\ \lambda_i^{ys}(av) \in R_+^{24},\ i \in \overline{1,m};$$

$$z_j^g = (z_j^g(1),\ldots,z_j^g(24));\ z_j^w = (z_j^w(1),\ldots,z_j^w(24));\ z_j^s = (z_j^s(1),\ldots,z_j^s(24));$$

$$p_j^z = (p_j^z(1),\ldots,p_j^z(24));\ \lambda_j^{zw}(av) = (\lambda_j^{zw},\ldots,\lambda_j^{zw});\ \lambda_j^{zs}(av) = (\lambda_j^{zs},\ldots,\lambda_j^{zs});$$

$$z_j^{st} = (z_j^{st}(1),\ldots,z_j^{st}(24));\ \lambda_j^{zw}(av),\ \lambda_j^{zs}(av) \in R_+^{24},\ j \in \overline{1,n};$$

$$u_k^g = (u_k^g(1),\ldots,u_k^g(24));\ u_k^w = (u_k^w(1),\ldots,u_k^w(24));\ u_k^s = (u_k^s(1),\ldots,u_k^s(24));$$

$$p_k^u = (p_k^u(1),\ldots,p_k^u(24));\ \lambda_k^{uw}(av) = (\lambda_k^{uw},\ldots,\lambda_k^{uw});\ \lambda_k^{us}(av) = (\lambda_k^{us},\ldots,\lambda_k^{us});$$

$$u_k^{st} = (u_k^{st}(1),\ldots,u_k^{st}(24));\ \lambda_k^{uw}(av),\ \lambda_k^{us}(av) \in R_+^{24},\ k \in \overline{1,r},$$

let $y^g = (y_1^g, y_2^g,\ldots,y_m^g)$, $z^g = (z_1^g, z_2^g,\ldots,z_n^g)$, $u^g = (u_1^g, u_2^g,\ldots,u_k^g)$, $p^y = (p_1^y, p_2^y,\ldots,p_m^y)$, $p^z = (p_1^z, p_2^z,\ldots,p_n^z)$, $p^u = (p_1^u, p_2^u,\ldots,p_k^u)$, and let

$$\tilde{y} = (y^g; y_1^w, y_2^w,\ldots,y_m^w; y_1^s, y_2^s,\ldots,y_m^s; y_1^{st}, y_2^{st},\ldots,y_m^{st}),$$

$$\tilde{p}^y = (p^y; 0,0,\ldots,0; 0,0,\ldots,0; 0,0,\ldots,0),$$

$$\tilde{q}^y = (0; \lambda_1^{yw}(av),\ldots,\lambda_m^{yw}(av); \lambda_1^{ys}(av),\ldots,\lambda_m^{ys}(av); \pi_1^y,\ldots,\pi_m^y),$$

$$\tilde{z} = (z^g; z_1^w, z_2^w,\ldots,z_n^w; z_1^s, z_2^s,\ldots,z_n^s; z_1^{st}, z_2^{st},\ldots,z_n^{st}),$$

$$\tilde{p}^z = (p^z; 0,0,\ldots,0; 0,0,\ldots,0; 0,0,\ldots,0),$$

$$\tilde{q}^z = (0; \lambda_1^{zw}(av),\ldots,\lambda_n^{zw}(av); \lambda_1^{zs}(av),\ldots,\lambda_n^{zs}(av); \pi_1^z,\ldots,\pi_n^z),$$

$$\tilde{u} = (u^g; u_1^w, u_2^w,\ldots,u_k^w; u_1^s, u_2^s,\ldots,u_k^s; u_1^{st}, u_2^{st},\ldots,u_k^{st}),$$

$$\tilde{p}^u = (p^u; 0,0,\ldots,0; 0,0,\ldots,0; 0,0,\ldots,0),$$

$$\tilde{q}^u = (0; \lambda_1^{uw}(av),\ldots,\lambda_k^{uw}(av); \lambda_1^{us}(av),\ldots,\lambda_k^{us}(av); \pi_1^u,\ldots,\pi_k^u),$$

so that $\langle y^g, p^y\rangle = \langle \tilde{y}, \tilde{p}^y\rangle, \langle z^g, p^z\rangle = \langle \tilde{z}, \tilde{p}^z\rangle, \langle u^g, p^u\rangle = \langle \tilde{u}, \tilde{p}^u\rangle,$ let $\tilde{\tilde{y}} = (\tilde{y}, \tilde{z}, \tilde{u})$, $\tilde{\tilde{x}} = (\tilde{p}^y, \tilde{p}^z, \tilde{p}^u)$, $\delta = (\tilde{q}^y, \tilde{q}^z, \tilde{q}^u)$, let $\tilde{\tilde{t}} = (\theta^y; 0,0,\ldots 0, \theta^z; 0,0,\ldots 0, \theta^u; 0,0,\ldots 0)$, $\tilde{\tilde{s}} = (s^y; 0,0,\ldots 0, s^z; 0,0,\ldots 0, s^u; 0,0,\ldots 0)$, where all the zero components of the vectors $\tilde{\tilde{t}}$ and $\tilde{\tilde{s}}$ correspond to the components of the vectors $\tilde{y}$, $\tilde{z}$, and $\tilde{u}$

$$(y_1^w, y_2^w,\ldots,y_m^w; y_1^s, y_2^s,\ldots,y_m^s; y_1^{st}, y_2^{st},\ldots,y_m^{st}),$$

$$(z_1^w, z_2^w,\ldots,z_n^w; z_1^s, z_2^s,\ldots,z_n^s; z_1^{st}, z_2^{st},\ldots,z_n^{st}),$$

$$(u_1^w, u_2^w,\ldots,u_k^w; u_1^s, u_2^s,\ldots,u_k^s; u_1^{st}, u_2^{st},\ldots,u_k^{st}),$$

respectively, and let $\hat{y}=(\tilde{\tilde{y}},Y^g)$, $\hat{x}=(\tilde{\tilde{x}},0_Y)$, $\hat{t}=(\tilde{\tilde{t}},0_Y)$, $\hat{s}=(\tilde{\tilde{s}},0_Y)$, $\hat{\Delta}=(\delta,0_Y)$, where $Y^g=(Y^g(1),\ldots,Y^g(24))$, $0_Y\in R^{24}$ is the zero vector in R^{24}.

Taking into account the above notation, the functioning of the generator can be described by the following system of constraints [1]

$$\left\langle \varepsilon, Y^g \right\rangle - \left(\left\langle \varepsilon^y, y^g \right\rangle + \left\langle \varepsilon^z, z^g \right\rangle + \left\langle \varepsilon^u, u^g \right\rangle \right) - \left\langle \varepsilon, MAX_{loss}(Y^g) \right\rangle = 0,$$

$$H_{\min} \le \left\langle \varepsilon, Y^g \right\rangle \le H_{\max},$$

$$\left\langle y^g, p^y \right\rangle + \left\langle z^g, p^z \right\rangle + \left\langle u^g, p^u \right\rangle - \left\langle \varepsilon, MAX_{expen}(Y^g) \right\rangle - \Psi(\hat{y}) \to \max_{(p^y,p^z,p^u)},$$

where

$$MAX_{loss}(Y^g) = \left(\max_{\lambda_1 \in \overline{1,\Lambda_1}} (a_{\lambda_1} + b_{\lambda_1} Y^g(1)), \cdots, \max_{\lambda_{24} \in \overline{1,\Lambda_{24}}} (a_{\lambda_{24}} + b_{\lambda_{24}} Y^g(24)) \right)$$

describe the losses in transmitting electricity via high voltage lines [1], $a_{\lambda_l} + b_{\lambda_l} Y^g(l)$ are linear functions of the variables $Y^g(l)$, $\lambda_l \in \overline{1,\Lambda_l}$, a_{λ_l}, $b_{\lambda_1} \in R^1$, $l \in \overline{1,24}$ (within a certain segment of the production volume $\lfloor c_{\min}, c_{\max} \rfloor$), ε, ε^y, ε^z, ε^u are vectors of corresponding dimensions with all the components equaling 1, and $H_{\min}$ and $H_{\max}$ are the minimal and the maximal technologically possible production capacities of the generator within the twenty-four hours, respectively. Here, the function $\Psi(\hat{y})$, describing the generator expenses associated with transmitting electric energy to the grid customers, has the form

$$\Psi(\hat{y}) = \theta \sum_{l=1}^{24} \max_{\lambda_l \in \overline{1,\Lambda_l}} (a^l_{\lambda_l} + b^l_{\lambda_l} Y^g(l)) + \left(\sum_{i=1}^{m} \left\langle \tilde{\varepsilon}^y, \tilde{y} \right\rangle_i \theta^y_i + \sum_{j=1}^{n} \left\langle \tilde{\varepsilon}^z, \tilde{z} \right\rangle_j \theta^z_j + \sum_{k=1}^{r} \left\langle \tilde{\varepsilon}^u, \tilde{u} \right\rangle_k \theta^u_k \right).$$

and

$$MAX_{expen}(Y^g) = \left(\max_{\mu_1 \in \overline{1,\Gamma_1}} (c_{\mu_1} + b_{\mu_1} Y^g(1)), \cdots, \max_{\mu_{24} \in \overline{1,\Gamma_{24}}} (c_{\mu_{24}} + b_{\mu_{24}} Y^g(24)) \right)$$

describe the generator expenses associated with producing electricity [1], where $c_{\mu_l} + b_{\mu_l} Y^g(l)$ are linear functions of the variables $Y^g(l)$, c_{μ_l}, $b_{\mu_l} \in R^1$, $l \in \overline{1,24}$ (within the above segment $[c_{\min}, c_{\max}]$), and it is assumed that the inclusions $(s^y_1,\cdots,s^y_m,s^z_1,\cdots,s^z_n,s^u_1,\cdots,s^u_k,0_Y) \in \hat{S}$ and $(\tilde{y},\tilde{z},\tilde{u},Y^d) \in \hat{\Omega}$, $(\tilde{p}^y,\tilde{p}^z,\tilde{p}^u 0_Y) \in \hat{M}$, $(\theta^y_1,\cdots,\theta^y_m,\theta^z_1,\cdots,\theta^z_n,\theta^u_1,\cdots,\theta^u_k,0_Y) \in \hat{T}$ hold for the variables describing the functioning of all the large grid customers, the generator, and the transmission company; here, $\tilde{\varepsilon}^y, \tilde{\varepsilon}^z, \tilde{\varepsilon}^u$ are vectors of corresponding dimensions with all the components equaling 1, and $\hat{M}$, $\hat{\Omega}$, $\hat{S}$ and $\hat{T}$ are some polyhedra.

As shown in [2], the interaction of the generator, all the large grid customers, and the transmission company can be described by the m+n+r+2-person game

$$(Game\ 1)\qquad \begin{array}{l} \langle \hat{y},\hat{x}\rangle - \langle \hat{y},\hat{t}\rangle - f_1(\hat{y}) - f_2(\hat{y}) \to \max\limits_{\hat{x}\in\hat{M}} \\ \langle \hat{y},\hat{t}\rangle + \langle \hat{y},\hat{s}\rangle + f_1(\hat{y}) \to \max\limits_{(\hat{t},\hat{s})\in\hat{T}\times\hat{S}}, \\ \langle \hat{y},\hat{x}\rangle_i + \langle \hat{\Delta},\hat{y}\rangle_i \to \min\limits_{\hat{y}\in\hat{\Omega}},\ i\in\overline{1,m}, \\ \langle \hat{y},\hat{x}\rangle_j + \langle \hat{\Delta},\hat{y}\rangle_j \to \min\limits_{\hat{y}\in\hat{\Omega}},\ j\in\overline{1,n}, \\ \langle \hat{y},\hat{x}\rangle_k + \langle \hat{\Delta},\hat{y}\rangle_k \to \min\limits_{\hat{y}\in\hat{\Omega}},\ k\in\overline{1,r}, \end{array}$$

where the polyhedra $\hat{\Omega}$, $\hat{M}$, $\hat{T}$ and $\hat{S}$ can be viewed as sets of the player strategies, and it is assumed that these polyhedra are described by compatible systems of linear inequalities (which include system (1) for industrial customer *i* and systems similar to (1) for utility company *j* and for group of advanced customers *k*), described in detail in [1]. Here $f_1(\hat{y}) = \langle \theta\varepsilon, MAX_{loss}(\hat{y})\rangle$, $f_2(\hat{y}) = \langle \varepsilon, MAX_{expen}(\hat{y})\rangle$, the scalar product $\langle \hat{y},\hat{x}\rangle_i$ is a part of the scalar product $\langle \hat{y},\hat{x}\rangle$ related to the variables associated with industrial customer i, $i\in\overline{1,m}$, $\varepsilon\in R_+^{24}$ is the vector with all the components equaling 1 , and the functions $MAX_{loss}(Y^g)$, and $MAX_{expen}(Y^g)$ can be viewed as those of the vector $\hat{y}$ due to the above relations between the vectors $\hat{y}$ and Y^g. Let 0^{yM}, 0^{zM}, 0^{uM} be zero vectors of the same dimensions as are the vectors y^M, z^M, and u^M, respectively, let

$$(yM)_i = ((yM)_i(1),\ldots,(yM)_i(24)),\ (zM)_j = ((zM)_j(1),\ldots,(zM)_j(24)),\ (uM)_k = ((uM)_k(1),\ldots,(uM)_k(24)),$$

$$y^M = ((yM)_1,\ldots,(yM)_m),\ z^M = ((zM)_1,\ldots,(zM)_n),\ u^M = ((uM)_1,\ldots,(uM)_k),$$

$$\hat{\hat{x}} = (\hat{x}, 0^{yM}, 0^{zM}, 0^{uM}),\ \hat{\hat{t}} = (\hat{t}, 0^{yM}, 0^{zM}, 0^{uM}),\ \hat{\hat{y}} = (\hat{y}, y^M, z^M, u^M),$$

$$i\in\overline{1,m},\ j\in\overline{1,n}, k\in\overline{1,r},$$

$c^{yM}(av)\in R^m$ be the vector whose component *i* is an average market price for a unit volume of electricity for industrial customer *i*, $i\in\overline{1,m}$,

$s^{yM}\in R^m$ be the vector whose component *i* is the price for transmitting a unit volume of electricity for industrial customer i, $i\in\overline{1,m}$ from the electricity markets,

and let $c^{zM}(av)\in R^n$, $s^{zM}\in R^n$ and $c^{uM}(av)\in R^k$, $s^{uM}\in R^k$ have the same meaning for utility company *j*, $j\in\overline{1,n}$ and for group of advanced customers *k*, $k\in\overline{1,r}$ as do $c^{yM}(av)$, s^{yM} for industrial customer *i*, $i\in\overline{1,m}$, respectively.

Further, let $\hat{\hat{\Delta}} = (\hat{\Delta}, c^{yM}(av), c^{zM}(av), c^{uM}(av))$, let s^{yM}, s^{zM}, s^{uM} be components of a vector from a polyhedron $P\subset\Pi_s^M$, let $\hat{\hat{M}} = \hat{M}\times(0^{yM}, 0^{zM}, 0^{uM})$, $\hat{\hat{\Omega}} = \hat{\Omega}\times Q$, where $Q\subset\Pi_y^M$ is a polyhedron reflecting the generator's assessment of the availability of electricity in the market (s) for buying by the large grid customers, where Π_s^M and Π_y^M are parallelepipeds in the spaces of corresponding dimensions, $\hat{\hat{S}} = (\hat{S}\times P)$, and $\hat{\hat{T}} = \hat{T}\times(0^{yM}, 0^{zM}, 0^{uM})$ so that $\hat{\hat{M}}$, $\hat{\hat{\Omega}}$, $\hat{\hat{S}}$ and $\hat{\hat{T}}$ are polyhedra described by linear constraints similar to those describing the polyhedra $\hat{M}$, $\hat{\Omega}$, $\hat{S}$ and $\hat{T}$, respectively, but binding the variables $\hat{\hat{x}}, \hat{\hat{y}}, \hat{\hat{s}}$, and $\hat{\hat{t}}$, where $\hat{\hat{s}} = (\hat{s}, s^{yM}, s^{zM}, s^{uM})$. Here, it is assumed that a) the hourly prices for transmitting to the gird customers electricity that is bought in the market (s) are constant within every hour during the twenty-four-hour period of time (though, they are, generally, different for each customer), and b) the generator has information only about the average prices at which the large grid customers can buy electricity in the markets.

3. Basic results

Let us consider an auxiliary three-person game in the form

$$(Game\ 2)\qquad \begin{aligned} &\left\langle \hat{\hat{y}}, \hat{\hat{x}} \right\rangle - \left\langle \hat{\hat{y}}, \hat{\hat{t}} \right\rangle - F_1(\hat{\hat{y}}) - F_2(\hat{\hat{y}}) \to \max_{\hat{\hat{x}} \in \hat{\hat{M}}}, \\ &\left\langle \hat{\hat{y}}, \hat{\hat{t}} \right\rangle + \left\langle \hat{\hat{y}}, \hat{\hat{s}} \right\rangle + F_1(\hat{\hat{y}}) \to \max_{(\hat{\hat{t}}, \hat{\hat{s}}) \in \hat{\hat{T}} \times \hat{\hat{S}}}, \\ &\left\langle \hat{\hat{y}}, \hat{\hat{x}} \right\rangle + \left\langle \hat{\hat{y}}, \hat{\hat{s}} \right\rangle + \left\langle \hat{\hat{\Delta}}, \hat{\hat{y}} \right\rangle \to \min_{\hat{\hat{y}} \in \hat{\hat{\Omega}}},. \end{aligned}$$

where the expressions for the functions F_1 and F_2 are the same as for the functions f_1 and f_2, and the equalities $F_1(\hat{\hat{y}}) = f_1(\hat{y})$ and $F_2(\hat{\hat{y}}) = f_2(\hat{y})$ hold.

Finally, let us consider the auxiliary two-person game on the polyhedra $\hat{\hat{\Omega}}$ and $\hat{\hat{M}} \times \hat{\hat{S}}$ of the player strategies

$$(Game\ 3)\qquad (\hat{\hat{y}}^*, (\hat{\hat{x}}^*, \hat{\hat{s}}^*)) \in Sp_{(\hat{\hat{y}}, (\hat{\hat{x}}, \hat{\hat{s}})) \in \hat{\hat{\Omega}} \times (\hat{\hat{M}} \times \hat{\hat{S}})} (\left\langle \hat{\hat{y}}, \hat{\hat{x}} + \hat{\hat{s}} \right\rangle + \left\langle \hat{\hat{\Delta}}, \hat{\hat{y}} \right\rangle),$$

where the payoff function is minimized with respect to $\hat{\hat{y}}$ and is maximized with respect to $(\hat{\hat{x}}, \hat{\hat{s}})$.

Assertion 1.

A quadruple of the vectors ($\hat{\hat{y}}^*$, $\hat{\hat{x}}^*$, $\hat{\hat{t}}^*$, $\hat{\hat{s}}^*$) is an equilibrium point in Game 2 if and only if the triple of the vectors ($\hat{\hat{y}}^*$, $(\hat{\hat{x}}^*, \hat{\hat{s}}^*)$) is a saddle point in Game 3, and the inclusion $\hat{\hat{t}}^* \in Arg \max_{\hat{\hat{t}} \in \hat{\hat{T}}} \left\langle \hat{\hat{y}}^*, \hat{\hat{t}} \right\rangle$ holds.

Assertion 2.

An equilibrium point in Game 2 determines an equilibrium point in Game 1.

Proofs of both assertions are practically identical to those of the corresponding two basic assertions presented in [2].

Remark. One can easily be certain that particular forms of the functions $F_1(\hat{\hat{y}})$ and $F_2(\hat{\hat{y}})$ do not affect the fact that each equilibrium point in Game 2 is an equilibrium in Game 1 [2] though, generally, the set of equilibrium points in Game 2 determines only a subset of the set of equilibrium points in Game 1, and the whole set of equilibrium points in Game 1 certainly depends on particular forms of both functions.

4. On calculating equilibrium points in Game 3

By setting $m_1 = n_1 = m$, where m is the dimension of the vectors $\hat{\hat{y}}$, $\hat{\hat{x}}$, $\hat{\hat{s}}$, by setting $p_1 = \hat{\hat{\Delta}},\ x_1 = \hat{\hat{y}},$ $y_1 = \hat{\hat{x}} + \hat{\hat{s}},\ q_1 = 0,$ and by setting $E_m = D_1,$ where E_m is the $m \times m$ matrix whose non-zero elements equal 1 and occupy the main diagonal of E_m, one can be certain that Game 3 is a two-person game with the payoff function $\langle p_1, x_1 \rangle + \langle x_1, D_1 y_1 \rangle + \langle q_1, y_1 \rangle$ where M_1 and Ω_1 are polyhedra described by compatible systems of linear inequalities $M_1 = \{x_1 \in R_+^{m_1} : A_1 x_1 \geq b_1\}$, $\Omega_1 = \{y_1 \in R_+^{n_1} : B_1 y_1 \geq d_1\}$, A_1, B_1, D_1 are matrices, and b_1, d_1, p_1, q_1, x_1, y_1 are vectors of corresponding dimensions and structures.

Theorem [4]. The solvability of Game 3 is equivalent to that of two linear programming problems

$$\begin{aligned} &\langle b_1, z_1 \rangle + \langle q_1, y_1 \rangle \to \max_{(z_1, y_1) \in Q_1}, \\ &\langle -d_1, t_1 \rangle + \langle p_1, x_1 \rangle \to \min_{(t_1, x_1) \in P_1}, \end{aligned} \tag{2}$$

forming a dual pair, where $Q_1 = \{(z_1, y_1) \geq 0 : z_1 A_1 \leq p_1 + D_1 y_1, B_1 y_1 \geq d_1\}$, $P_1 = \{(t_1, x_1) \geq 0 : t_1 B_1 \leq -q_1 - x_1 D_1, A_1 x_1 \geq b_1\}$, and t_1, z_1 are vectors of corresponding dimensions, which allows one to find equilibrium points in a solvable Game 3 by linear programming techniques for any imaginable numbers of constraints and variables that may appear in practical problems.

5. Concluding Remarks

1. Each equilibrium point in Game 1 (and there may be more than one equilibrium point in this game) determines the whole spectrum of parameters that all the large grid customers, the generator, and the transmission company are interested in estimating in an attempt to analyze financial strategies available to them [1], [2]. For instance, for the generator, each equilibrium point determines, in particular, a) an optimal hourly production volume of electricity, and b) a set of optimal (for the generator) hourly prices for electricity to be paid to the generator by the large grid customers. As shown in [1], [2], it is easy to modify constraints describing the set of the generator strategies in Game 1 to find its optimal constant hourly production within any twenty-four-hour period of time.

However, the description of the interaction of the above players in the form of Game 1 reflects a certain simplification of reality, since this game does not take into consideration how the transmission company can affect the equilibrium volumes and prices (in the potential contractual agreements among the players) by choosing the prices at which it charges the generator for transmitting electricity [1]. Yet, Game 1 correctly covers at least two important practical cases: a) when only the grid customers are charged for the electricity transmitted to them (i.e., when the equality $\widehat{t} = 0$ holds), and b) when the generator assumes that in its potential contractual agreements with the customers, the customers would agree that the transmission company can choose any prices for transmitting electricity to the customers (which may differ from those for the generator) as long as the vectors of these prices belong to the same polyhedron, i., e., the equality $\widehat{T} = \widehat{S}$ holds.

One should emphasize that the description of the interaction of the generator, the large grid customers, and the transmission company that takes into consideration the above dependencies of the equilibrium volume and prices on the vector $\widehat{t} \in \widehat{T}$ leads to studying games with a more complicated structure than that of Game 1; however, the analysis of such games lies beyond the scope of the present article.

2. Though Games 1-3 were formulated from the generator perspectives, to be competitive, each large grid customer and the transmission company should be interested in analyzing their own strategies by solving the same games (though, possibly, with their own data and their own estimates of parameters in the models underlying the description of the functioning of the game players).

3. The demonstrated possibility to calculate equilibrium points in Game 1 by solving linear programming problems (though under assumptions underlying the linear structure of the constraints describing the sets of player strategies in Game 1) seems promising for the use of game theory approaches and models in solving large-scale quantitative management problems being of national importance for every country.

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